Study Material – Sem. 6 – C13T – Polarization of Electromagnetic Waves – Dr. T. Kar – Class 3

13.20 ANALYSIS OF CIRCULARLY POLARISED LIGHT BY USING QUARTER-WAVE PLATE :

Fast and slow axes of $\lambda/4$ plate :

A quarter-wave plate ($\lambda/4$ plate) has two principal directions, one parallel and another perpendicular to the optic axis which is lying on its suface. The vibrations of the *E*-ray are along the optic axis while those of *O*-ray are perpendicular to the optic axis. In the case of a positive crystal like quartz, the *O*-ray travels faster than *E*-ray in the direction perpendicular to the optic axis and it is customary to call the direction perpendicular to the optic axis of positive crystal as the *fast axis* while the axis of positive crystal as the *slow axis*. In the case of a negative crystal like calcite, *O*-rays travel slower than *E*-ray in the direction perpendicular to the optic axis and for this the axis perpendicular to the optic axis of a negative crystal is called slow axis while the axis of the negative crystal is called fast axis.

Principle of analysis :

Suppose a right-handed circularly polarised light represented at $z = 0$ by $x = a\cos\omega t$, $y = a\cos(\omega t + \pi/2)$ is made incident normally on the $\lambda/4$ plate (made of $+ve$ crystal) whose fast axis is vertical and slow axis is horizontal (Fig. 13.20-1). This circular vibration on incidence, will be resolved into two rectangular components in which the phase of *y*-component will be in advance of that of *x*-component by $\pi/2$. During their passage through the $\lambda/4$ plate there will be an additional phase advancement of *y*-component relative to *x*-component by $\pi/2$. So the emergent light will combine to form a linear vibration R_1R_2 making an angle $-45°$ with the slow axis;

$$x = a\cos\omega t,\ y = a\cos(\omega t + \pi) = -a\cos\omega t, \text{ or, } y = -x$$

Hence to cut off this vibration, the principal section of analysing Nicol should be along N_1N_2 which makes an angle of $+45°$ with the slow axis (Fig. 13.20-1).

If left-handed circularly polarised light represented at $z = 0$ by $x = a\cos(\omega t + \pi/2)$, $y = a\cos\omega t$ is made to fall on this $\lambda/4$ plate normally, then it will be resolved into two rectangular components, in which the phase of *x*-component will exceed that of *y*-component by $\pi/2$. On passing through the $\lambda/4$ plate there will be a phase advancement of y-component relative to the x-component by $\pi/2$. So the emergent light can be represented by,

$$x = a\cos(\omega t + \pi/2), \quad y = a\cos(\omega t + \pi/2) \quad \text{or,} \quad y = x.$$

It represents a linear vibration along R_1R_2 making an angle of $+45°$ with the slow axis (Fig. 13.20-2). To cut off this vibration the principal section of the analysing Nicol should be along N_1N_2 which makes an angle of $-45°$ with the slow axis.

Experiment :

In the experimental arrangement a parallel beam of circularly polarised light falls normally on the quarter-wave plate whose slow axis is kept horizontal. The analysing Nicol A, placed behind the $\lambda/4$ plate, is now rotated to make the field dark. If the principal section of analyser makes an angle of $+45°$ with the slow axis of the $\lambda/4$ plate then the

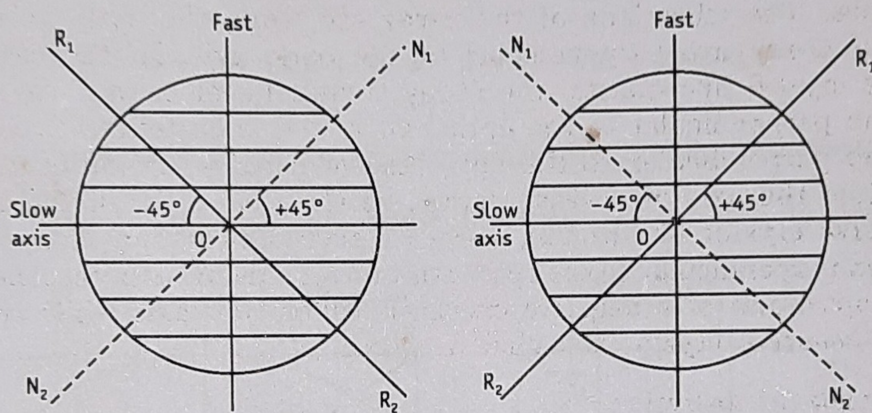

Fig. 13.20-1 Fig. 13.20-2

given circularly polarised light is right-handed (Fig. 13.20-1). On the other hand, if the shorter diagonal of analyser (A) makes an angle of $-45°$ with the slow axis of $\lambda/4$ plate then the circularly polarised light is left-handed (Fig. 13.20-2).

13.21 ANALYSIS OF ELLIPTICALLY POLARISED LIGHT BY QUARTER-WAVE PLATE :

Principle of analysis :

If an elliptically polarised light be made incident normally on a quarter-wave plate so that the axes of the elliptical vibration may be coincident with the principal directions of $\lambda/4$ plate then the elliptic vibration at incidence will be resolved into two rectangular vibrations having a phase difference of $\pi/2$. The $\lambda/4$ plate will introduce a further relative phase change of $\pi/2$. Hence the phase difference of the two rectangular vibrations emerging from the $\lambda/4$ plate will be either zero or π. In both cases the resultant vibration will be linear which can be cut off by the analysing Nicol A.

When this happens, the principal directions of $\lambda/4$ plate will represent the positions of the axes of elliptic vibrations. If the principal section of the analysing Nicol (A) makes an angle θ with any one of the two principal directions of $\lambda/4$ plate (at the position of extinction) then $\tan\theta$ = ratio of the amplitudes of two rectangular vibrations forming elliptically polarised light.

Suppose a left-handed elliptical vibration is incident normally on $\lambda/4$ plate such that the axes of elliptic vibration coincide with the axes of the $\lambda/4$ plate. The incident wave then can be resolved into two linear vibrations differing in phase by $\pi/2$. Let

$$x = a\cos(\omega t + \pi/2), \quad y = b\cos\omega t$$

where x-axis is along the optic axis and y-axis is perpendicular to it. Now, if we assume a $+ve$ crystal then $v_o > v_e$ i.e., there will be a phase advancement of y-component (O-ray) by $\pi/2$ relative to the x-component. So the emergent vibrations become

$$x = a\cos(\omega t + \pi/2), \quad y = b\cos(\omega t + \pi/2) \quad \text{or,} \quad y = \frac{b}{a}x$$

It represents a linear vibration making an angle $\theta = \tan^{-1}\dfrac{b}{a}$ with the x-axis. It can be extinguished when the principal section of the

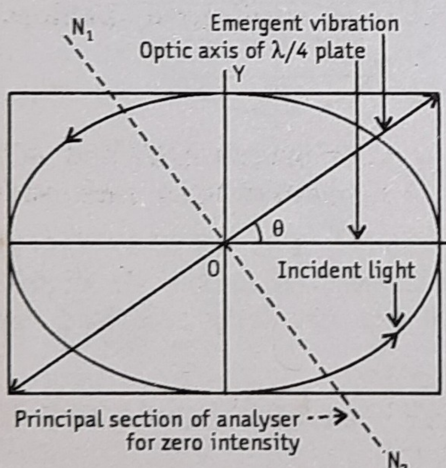

Fig. 13.21-1

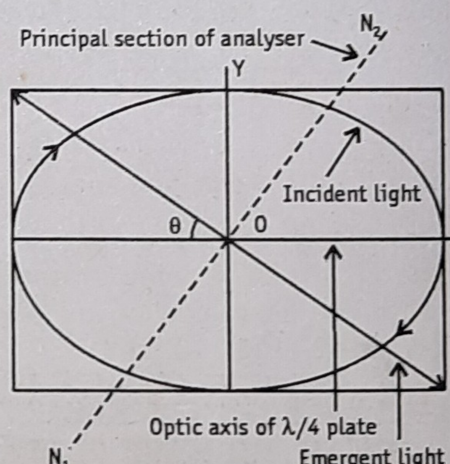

Fig. 13.21-2

analyser is perpendicular to the plane of vibration of the emergent light (Fig. 13.21-1).

If the incident light is right-handed elliptic vibration, then incident light can be resolved as,

$$x = a\cos\omega t, \quad y = b\cos(\omega t + \pi/2)$$

The emergent vibrations become

$$x = a\cos\omega t, \quad y = b\cos(\omega t + \pi) = -b\cos\omega t$$

Therefore, $$y = -\frac{b}{a}x$$

which represents a linear vibration making an angle $\theta = -\tan^{-1}\frac{b}{a}$ with the x-axis (Fig. 13.21-2).

Experiment :

A parallel beam of elliptically polarised light is incident normally on a $\lambda/4$ plate. The analysing Nicol is placed behind the $\lambda/4$ plate to receive the transmitted light. The $\lambda/4$ plate is now rotated in small steps and at each step the analyser is rotated to the position of minimum intensity. Proceeding in this way we get zero intensity for one position of the analyser. This time the principal directions of $\lambda/4$ plate will coincide with the axes of elliptical vibration.

The tangent of the angle between the principal section of the analyser and the axes of $\lambda/4$ plate will give the ratio of the semi-axes of the elliptical vibration.

13.22 BABINET'S COMPENSATOR :

In the study of optical phenomenon it is found to be convenient to have a crystal plate of variable thickness. Babinet's compensator is such a plate. The use of $\lambda/4$ plate in the production and analysis of elliptically polarised light is limited to a narrow range of wavelength. Babinet's compensator has no such limitation.

Construction :

It consists of two slender right-angled quartz prisms ABD and ACD placed together with their hypotenuse AD in contact with each other

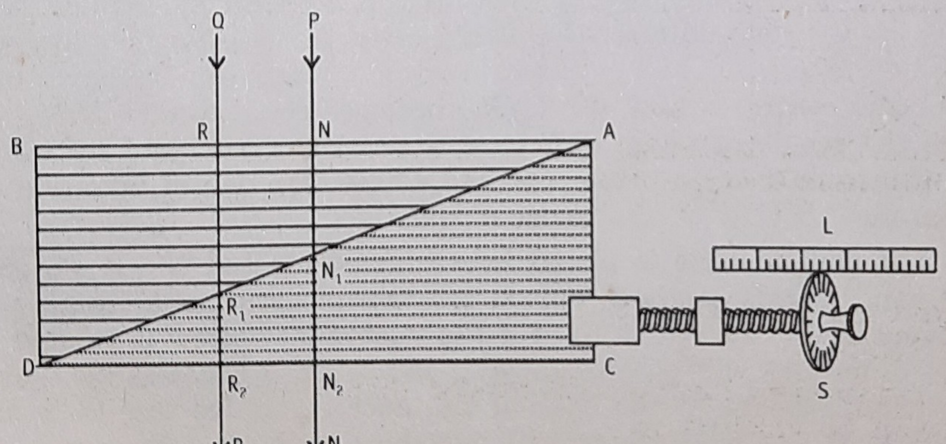

Fig.13.22-1

so that they together form a rectangular slab (Fig. 13.22-1). The axis of ABD is on the face AB and parallel to AB of the section ABD while the axis of *ACD* is on the face *CD* but perpendicular to the section *ACD*. Thus the axes of the two prisms are at right angles to each other so that the ordinary ray in one will behave as the extraordinary ray, when travelling in the other and *vice versa*. The thickness of the prisms are very small so that the separation between the ordinary and extraordinary rays is practically nil.

In the Jamin's modified form one of the prisms is shifted by a micrometer screw (Fig. 13.22-1).

Action :

Let a ray *PN* of plane polarised light be made incident on the face *AB* normally so that its plane of vibration is making an angle 45° with the optic axes. This ray will be broken up into extraordinary and ordinary rays whose vibration will respectively be parallel and perpendicular to the axis of *ABD*. If these two rays traverse the thickness d of the prism *ABD* then the path retardation introduced between these two rays by the prism *ABD* is $d(n_e - n_o)$ for quartz has a greater refractive index (n_e) for extraordinary ray than that (n_o) for ordinary ray. These two rays on entering the prism *ACD* will change their character, i.e., the ordinary and the extraordinary rays in the prism *ABD* will respectively behave as the extraordinary and the ordinary rays in the prism *ACD*. If these two rays traverse the thickness d' of the prism *ACD* then the path retardation introduced between these two rays by the prism *ACD* will be, $-d'(n_e - n_o)$, for here opposite phase will be introduced. Thus the resultant phase difference between the rays emerging from the two prisms is,

$$(d - d')(n_e - n_o) \qquad \text{...(13.22-1)}$$

and phase difference,

$$\delta = \frac{2\pi}{\lambda}(d - d')(n_e - n_o) \qquad \text{...(13.22-2)}$$

For the central region, d $(= NN_1)$ becomes equal to $d'(= N_1N_2)$ and hence the resultant phase difference is zero. The emergent light N_2N_3 will, therefore, remain plane polarised whose direction of vibration will be parallel to that of the incident light.

If the compensator is placed between two crossed Nicols we get a dark band in this position. This is the central dark band. Keeping the positions of crossed Nicols fixed if the micrometer screw is rotated the phase difference will go on changing and when δ becomes 2π again a dark band appears. If the shift of the micrometer for this is $2b$ then a shift x of the micrometer will introduce a phase difference

$$\frac{2\pi}{2b} \times x = \frac{\pi x}{b}$$

Thus by shifting one prism with help of a micrometer screw any amount of phase difference can be introduced between E-rays and O-rays. On the other hand, if any phase difference exists between E-ray and O-ray (for example, due to transmission through any doubly refracting crystal plate), it can be neutralised or compensated by the Babinet's compensator. This is why it is called a compensator.

Applications of compensator :

(a) To produce elliptically and circularly polarised light :

The prism (ACD) is shifted by $x = b/2$ from its position of coincidence with the prism ABD. As a result it will introduce a phase difference of $\pi/2$. Now if the direction of vibration of the incident plane polarised light (from a polariser) makes an angle α, which is not 45°, with the optic axis of the prism ABD, then the emergent light consisting of two rectangular vibrations, of unequal amplitude but differing in phase by $\pi/2$, will be elliptically polarised. On the other hand, if the value of α is 45°, the light emerging from the compensator will be circularly polarised, for the amplitudes of two rectangular components would be equal.

(b) To analyse an elliptically polarised light :

(*i*) *Determination of the phase difference between two vibrations forming elliptically polarised light :*

At first the compensator is placed between two crossed Nicols. By using a white light and rotating the micrometer screw the central dark band is made to coincide with a cross-wire fitted with an observing eye-piece. The polarising Nicol is then removed and the compensator is illuminated by the elliptically polarised light under test. The central dark band will be shifted. Micrometer screw head is now shifted through x till the central dark band again coincides with the cross-wire. In this way the compensator compensates the phase difference ϕ between the two components of the elliptic vibration. Thus,

$$\phi = \frac{\pi x}{b}$$

where b is such that a shift of micrometer head by $2b$ introduces a phase shift of 2π.

(*ii*) *Determination of the positions of the axes and the ratio of axes of elliptic vibration :*

At first the compensator is placed between two crossed Nicols. By using a white light and rotating the micrometer screw the central dark band is made to coincide with a cross-wire fitted with an observing eye-piece. The micrometer screw is then shifted by $b/2$ to introduce a phase difference of $\pi/2$ at the centre. The polariser is now removed and the compensator is illuminated by the elliptically polarised light under test.

The compensator is then rotated by small steps and at each step the analyser is rotated to the position of minimum intensity. The process is repeated until the central dark band becomes again coincident with the cross-wire. This time the axes of the two prisms will be parallel to the axes of elliptic vibration. Thus the positions of the axes of elliptic vibration are found out.

At this position the incident elliptically polarised light can be resolved into two components parallel to the axes of the two prisms, having a phase difference of $\pi/2$. The compensator introduces an additional phase difference of $\pi/2$ at the central region. Thus total phase difference becomes either 0 or π and hence the two emerging vibrations combine to form a linear vibration. Accordingly the emergent vibration can be extinguished by the analysing Nicol. In this position the tangent of the angle (θ) which the principal section of the analyser makes with the optic axis of the quartz prisms gives the ratio of axes of the elliptic vibration (Figs. 13.21-1 and 13.21-2).

13.23 OPTICAL ACTIVITY :

If a plane polarised light is passed through some crystals like quartz along the optic axis the plane of vibration gradually undergoes rotation about the optic axis. The angle of rotation is found to depend on the thickness and nature of the crystal and also on the wavelength of the light employed. This phenomenon is called **optical activity** or **rotatory polarisation** and the substance which rotates the direction of vibration of the incident polarised light is called *optically active substance.*

There are two classes of active substances. One class of substance rotates the line of vibration of the incident light towards right and the substances belonging to this class are called **dextro-rotatory** substances. Another class of substance rotates the line of vibration towards left and the substances which belong to this class are called **laevo-rotatory** substances.

The optical activity of crystals is associated with the structural dissymmetry of the crystals. Optically active crystals are found to possess one or more screw axis of symmetry. Here the atoms or molecules constituting the crystal are arranged on a spiral which may be left-handed or right-handed. Some crystals occur in both the varieties, one of which is the mirror image of the other producing rotation in opposite directions. Some substances in solution show this property. In this case optical activity is associated with the asymmetrical structure of the molecules themselves.There may be a preferential direction of rotation associated with a molecule. Though molecules in a solution are randomly oriented the rotations do not cancel out because the sense of rotation is same for molecules having same type of dissymmetry. Thus optical activity is caused by structural dissymmetry of crystals or by molecular dissymmetry. There are substances in which optical activity is associated with both kinds of dissymmetry.

Demonstration :

If we take two crossed Nicols whose principal planes are at right angles to each other, then the field of the analysing Nicol would be dark. If now a quartz crystal cut perpendicular to its optic axis is put between the crossed Nicols, then the field of the analyser at once becomes bright. This shows that the direction of vibration of the polarised light incident on the quartz, after transmission, has rotated by a certain angle and has taken up a new direction of vibration. If the analyser is now rotated by a certain angle then the field becomes dark again. This angle of rotation of the analyser is a measure of the rotation of the plane of polarisation by quartz crystal.

13.24 BIOT'S LAWS : MEANING OF SPECIFIC ROTATION, MOLECULAR ROTATION AND ROTATORY DISPERSION :

Biot made a systematic study of the natural rotation of the plane of polarisation by different substances, and established the following laws :-

(1) The angle (θ) of rotation produced by an active substance is proportional to the length (l) of the substance traversed by the ray and is not affected by turning the substance around, consequently the rotation will be annulled if the ray is reflected back through the substance.

(2) The combined rotation produced by two different substances having different thickness is the algebraic sum of the rotation caused by each separately.

(3) The amount of rotation (θ) in the case of a solution, is proportional to the amount of active substance (m) present per c.c. of the solution.

(4) The amount of rotation varies with the wavelength of the incident light and with temperature of the substance. The rotation is approximately proportional to the inverse square of the wavelength.

In case of a solution, the first and third laws may be combined to have the relation, $\theta = slm$, where s is a constant and is called **rotatory power** or **specific rotation** of the solution. Thus specific rotation of a solution is defined as the amount of rotation produced by a 10cm column of liquid containing 1gm of the active substance per c.c. of the solution. Thus for a solution containing m mg. of active substance per c.c. the rotation (θ) for a path length l cm is given by,

$$\theta = s.m.(l/10) \quad \ldots(13.24\text{-}1)$$

where s is the specific rotation. For a pure liquid m is simply the density.

In the case of optically active crystals sepcific rotation is defined as the rotation produced by a crystal plate of thickness 1mm.

Molecular rotation is obtained by taking the product of the specific rotation and the molecular weight of the substance. According to the fourth law the angle of rotation of the plane of vibration of a plane polarised light produced by an optically active substance is found to be approximately proportional to the inverse square of the wavelength. So if a plane polarised white light is passed through an optically active substance it will decompose into constituent colours. This phenomenon is known as **rotatory dispersion.**